Permutation and Combination:

Class 1:

Fundamental Principle of counting:

1st thing : m ways

2nd thing : n ways

Two things can be done together = m x n

Concept : "and" : x : multiply

Concept : "or" : + : addition

Combination : Selection Only :

The number of combinations of n different things taken r at a time

= nCr

= n! / r! X (n-r)!

Example : Select 2 things out of 7 different things

7C2 = 7! / 2! X 5!

= 7 x 6 x 5! / 2! X 5!

= 7 x 6 / 1 x 2

= 21 ways

7C2 = 7 x 6 / 1 x 2 = 21 ways

10C3 = 10 x 9 x 8 / 1 x 2 x 3

= 5 x 3 x 8 = 120 ways

Rules :

1. nC0 = 1 way
2. nCn = 1 way
3. nC1 = n ways
4. nCr = nCn-r

Example : 15C11 = 15C(15-11)

= 15C4

Example 1 : A team of 3 boys and 2 girls has to be selected out of 5 boys and 4 girls. How many different ways are possible?

5C3 and 4C2 = 5C3 x 4C2

= (5 x 4 / 1 x 2)  x (4 x 3 / 1 x 2)

= 10 ways x 6 ways = 60 ways

Example 2 : 2 numbers are to be selected out of the first 100 natural numbers. In how many ways can this be done such that one is a prime while the other is a composite number?

There are 25 prime numbers from 1 to 100.

There are 75-1 ie 74 composite numbers.

25C1 x 74C1 = 25 x 74

Example 3 : 2 numbers are to be selected out of the first 100 natural numbers. In how many ways can this be done such that the numbers are divisible by 3 or 4 or 5?

By 3 : 33 numbers

By 4 : 25 numbers

By 5 : 20 numbers

By 12 : (LCM of 3 and 4) : 8

By 15 : (LCM of 3 and 5) : 6

By 20 : (LCM of 4 and 5) : 5

By 60 : (LCM of 3,4 and 5) : 1

Total numbers available

= 33+25+20-8-6-5+1

= 60 numbers

Answer : 60C2 = 60 x 59 / 1 x 2

Some Important applications :

1. Number of handshakes / Number of matches :

If there are n people in a room and each person shakes hands with everyone else, the number of handshakes = nC2

12C2 = 12 x 11 / 2

= 66 matches

Each person will shake hands with the remaining (n-1) other people

[n x (n-1)] / 2 = nC2

1. If there are n points in a plane and no 3 points are collinear, the number of straight lines that can be drawn = nC2

1. What if m points (m<n and m>2) are collinear and already on one straight line.

nC2-mC2+1

Class 2

Example: There are 7 points in a plane and 4 of them are already on a straight line. How many different straight lines can be formed using these points?

7C2 –4C2 +1 = 21-6+1

= 15+1 = 16

Alternate Method:

n-mC2 +mx(n-m) + 1

3C2 + 4 x 3 + 1

= 3 + 12 + 1

= 16

1. If a polygon has n vertices, then the number of diagonals will be equal to = nC2-n

Number of straight lines = nC2

But n out of those straight lines will be the side of the polygon.

The remaining lines will form the diagonal.

Triangle: 0 diagonals

3C2 = 3 lines

Rectangle: 2 diagonals

4C2 = 6 lines

4 side

Diagonals = 6-4 = 2

Decagon : 10 sides

Number of straight lines = 10C2

= (10 x 9) / 2 = 45

Number of sides = 10

Number of diagonals = 45-10 = 35

1. I have a 5 x 5 grid as shown in the figure:

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1. Number of rectangles :

A 5 x 5 grid has got 6 horizontal lines and 6 vertical lines.

To define a rectangle, we need to have 2 horizontal lines and 2 vertical lines.

6C2 x 6C2 = 15 x 15

= 225 rectangles

1. Number of squares:

1^2 + 2^2 + 3^2 + 4^2 + 5^2

= 1+4+9+16+25

= 55 squares

Example: There are n people standing on a circle. Each person forms a team with everyone else excluding his neighbours and performs for 3 minutes per team.

If the total time taken for all such performances is 42 minutes, find n?

Number of teams = 42/3 = 14

nC2 – n = 14

n(n-1)/2 - n = 14

n^2-n-2n = 28

n^2-3n = 28

n(n-3) = 28=======

Alternate:

n x (n-3) / 2    = 14

n(n-3) = 14

Concept: The total number of combinations of n different things taken 0 or some or all at a time:

nC0+nC1+nC2+nC3+….......nCn

This is the binomial expansion of

(1+1)^n = 2^n

If the cardinal number of a set ie number of distinct elements is n, then the number of subsets = 2^n

Identical things: The total number of combinations of n identical things taken 0 or some or all at a time

= (n+1) ways